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OXFORD

Parameterized Algorithms and Complexity for Function Merging with Branch Reordering

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Jun 19th, 2026

Code size optimization

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Applications: mobile apps, embedded systems.

Function merging

Function Merging

Input: Two functions F_1 and F_2.

Output: Merged function $F_\star$ such that:

- $F_\star$ captures both F_1 and F_2, and
- $F_\star$ is as small as possible.

Function merging (ex.)

Simplifying assumption: size $=$ # lines.

Function merging (ex.)

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1  def F1(x):
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```
13 def F2(x):
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```
19 def F1(x):
20   F⋆(x, 1)
```

```
21 def F2(x):
22   F⋆(x, 2)
```

Function merging (ex.)

23 lines $\rightarrow$ 22 lines!

Function merging via sequence alignment

SOTA approach: **sequence alignment** [CGO'19, PLDI'20].

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- View F_1 and F_2 as **strings of instructions**.
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More aligned instructions $\Rightarrow$ smaller $F_\star$.

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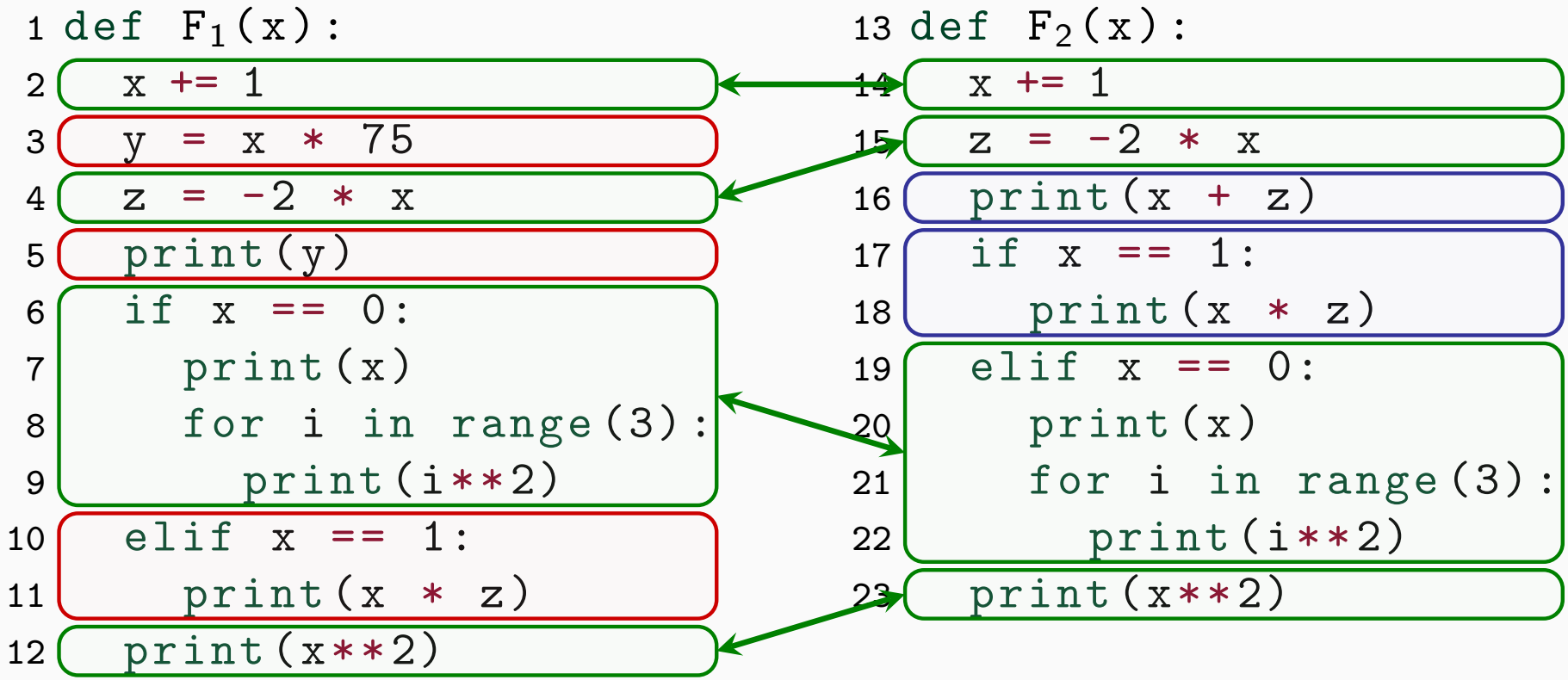

Function merging via sequence alignment (ex.)

```
1  def F*(x, b):
2    x += 1
3    if b == 1:
4      y = x * 75
5    z = -2 * x
6    if b == 1:
7      print(y)
8    else:
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Input: Two strings S_1 and S_2.

Output: Optimal sequence alignment of S_1 and S_2.

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$\texttt{FunctionMerging}(F_1, F_2) \Rightarrow \texttt{SequenceAlignment}(F_1, F_2)$.

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Textbook dynamic programming for Sequence Alignment
$\Longrightarrow \mathcal{O}(|F_1| \cdot |F_2|)$-time algorithm.

Sequence alignment + branch reordering

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$$\texttt{FunctionMerging}(F_1, F_2)$$

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$$\max_{\substack{F_1' \in \mathit{Reorder}(F_1) \\ F_2' \in \mathit{Reorder}(F_2)}} \texttt{SequenceAlignment}(F_1', F_2')$$

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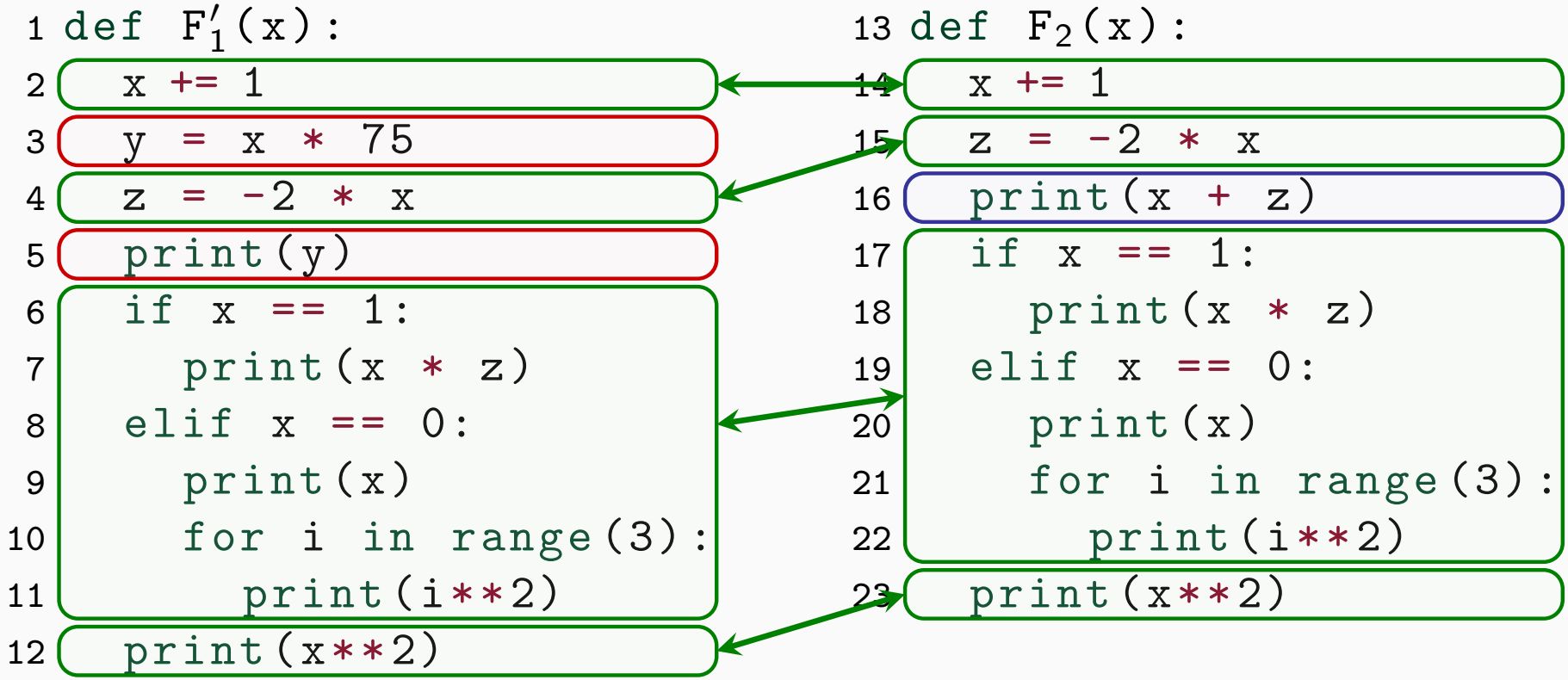

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23 lines $\rightarrow$ 22 lines $\rightarrow$ 20 lines!

Problem statement

Function Merging with Branch Reordering

Input: Two functions F_1 and F_2.

Output: Compute

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We study **algorithms and complexity** of this problem.

Complexity

Theorem (Hardness)

Function Merging with Branch Reordering is NP-hard.

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Identify some **parameters** of the input such that:

(1) On inputs of interest, these parameters are **restricted**.

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Satisfying (1) and (2) $\Rightarrow$ profit!

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Branching factor of $F_1 = 2$.

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Hypothesis: These parameters are typically small.

Two parameterized algorithms

Four parameters:

- branching factor of F_1/F_2 (b_1/b_2),
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Theorem (Restricting b_1, b_2, d_1, d_2)

Function Merging with Branch Reordering can be solved in

$$2^{\mathcal{O}(b_1 d_1 + b_2 d_2)} \cdot |F_1| \cdot |F_2| \text{ time.}$$

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For most of them, the hardness theorem applies.

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Open: what if b_1, b_2 are restricted, but d_1, d_2 are unbounded?

Overview of the first algorithm

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Input: Two functions F_1 and F_2.

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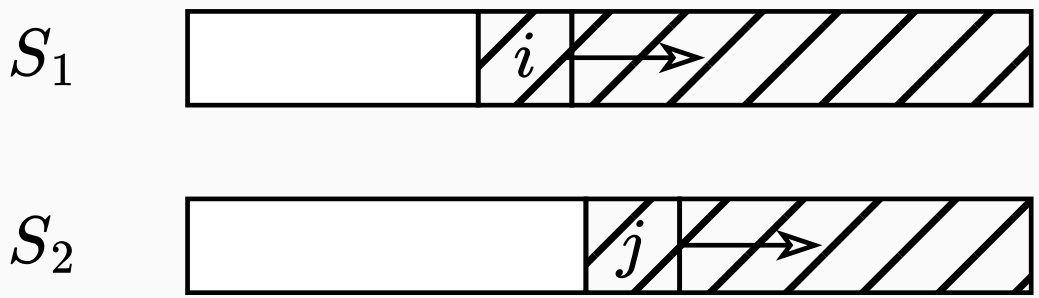
$$dp[i, j] = \texttt{SequenceAlignment}(S_1[i \ldots], S_2[j \ldots])$$



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dp algorithm for `SequenceAlignment`(S_1, S_2)

$\approx$

incremental, **simultaneous** scanning of S_1 and S_2.

$\Downarrow$

incremental, **simultaneous** exploration of $\mathit{Reorder}(F_1)$ and $\mathit{Reorder}(F_2)$; compute sequence alignment **on the fly**.

Programs to branching graphs

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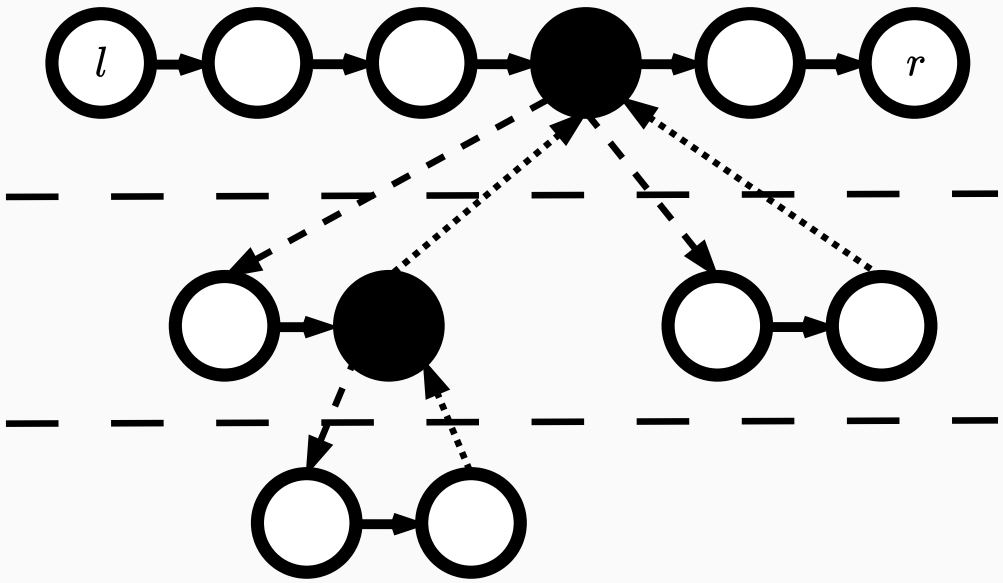

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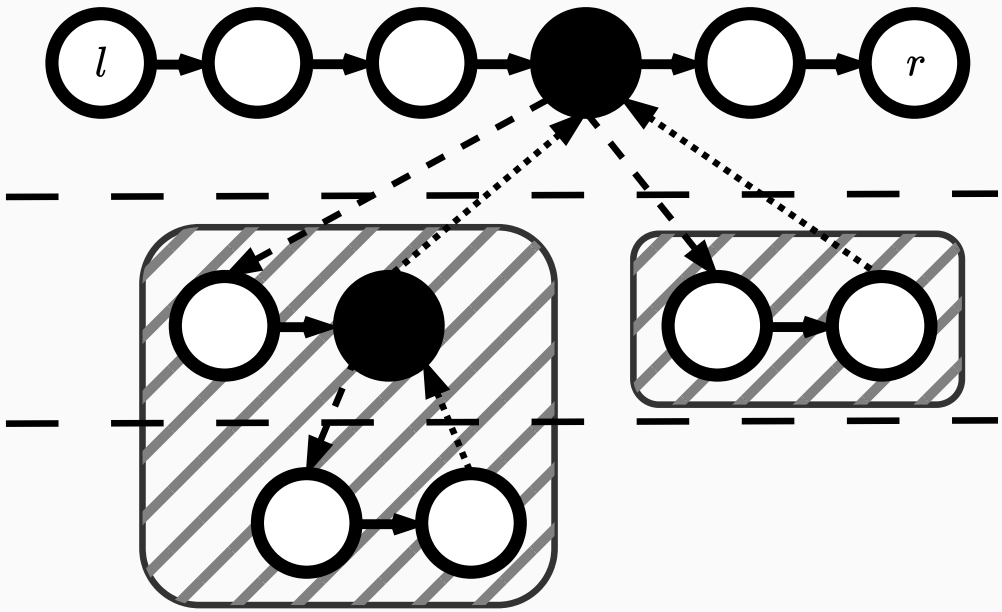
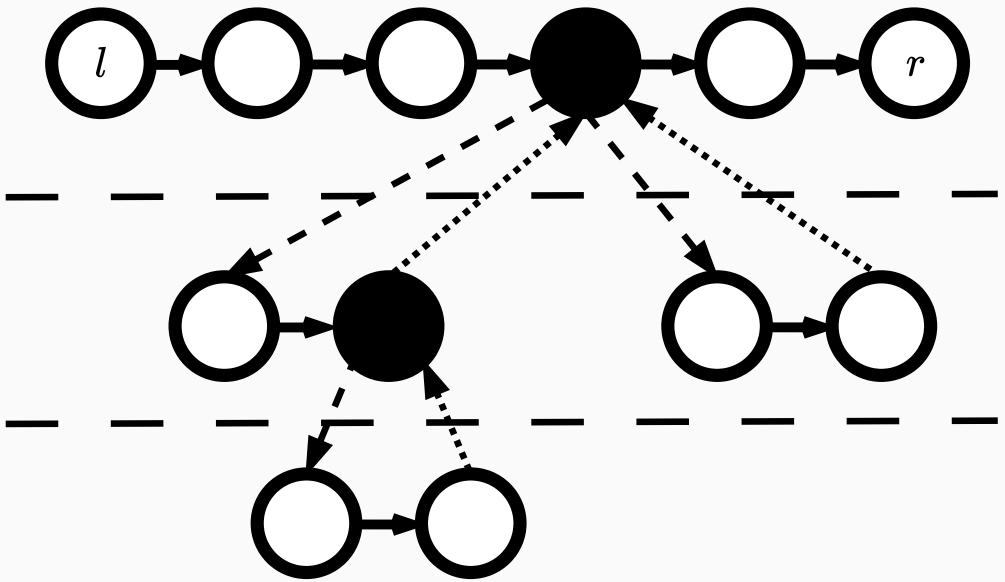


Exploring *Reorder*(*F*)



$\textsc{Explore}(\ell, \emptyset, \varepsilon)$.

procedure $\textsc{Explore}(u, X, S)$

if X contains all nodes: **print** S; **return.**

$X \leftarrow X \cup \{u\}$.

if u is ○: $\textsc{Explore}(\mathit{next}(u),\ X,\ S \cdot \mathit{instr}(u))$.

if u is ●: **forall** $v \in \mathit{branches}(u) \setminus X$: $\textsc{Explore}(v, X, S)$.

Exploring *Reorder*(F)

EXPLORE($\ell, \emptyset, \varepsilon$).

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  if u is ●: **forall** $v \in \mathit{branches}(u) \setminus X$: EXPLORE($v, X, S$).

Lemma (Compactness)

The number of distinct (u, X) *encountered in* EXPLORE *is at most*

$$2^{\mathcal{O}(bd)} \cdot |F|.$$

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$$\textsc{Explore}(u, X, \dots)$$

$$\Downarrow$$

$$\textsc{FunctionMerging}(u_1, X_1, u_2, X_2)$$

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\# dynamic programming states: $2^{\mathcal{O}(b_1 d_1 + b_2 d_2)} \cdot |F_1| \cdot |F_2|$.

Work per state: $\text{poly}(b_1, b_2, d_1, d_2) \cdot (|F_1| + |F_2|)$.

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Optimized $X_1, X_2 \Rightarrow \text{poly}(b_1, b_2, d_1, d_2)$ work per state.

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$$\textsc{FunctionMerging}(u_1, X_1, u_2, X_2)$$

\# dynamic programming states: $2^{\mathcal{O}(b_1 d_1 + b_2 d_2)} \cdot |F_1| \cdot |F_2|$.

Work per state: $\text{poly}(b_1, b_2, d_1, d_2) \cdot (|F_1| + |F_2|)$.

Optimized $X_1, X_2 \Rightarrow \text{poly}(b_1, b_2, d_1, d_2)$ work per state.

Theorem (Restricting b_1, b_2, d_1, d_2)

Function Merging with Branch Reordering can be solved in

$$2^{\mathcal{O}(b_1 d_1 + b_2 d_2)} \cdot |F_1| \cdot |F_2| \text{ time.}$$

Overview of the second algorithm

Theorem (Restricting b_1, b_2, d_1)

Function Merging with Branch Reordering can be solved in

$$2^{\mathcal{O}(b_1 d_1 + b_2)} \cdot (|F_1| \cdot |F_2|)^4 \text{ time.}$$

View first algorithm as

weighted reachability on a finite graph

of size $2^{\mathcal{O}(b_1 d_1 + b_2 d_2)} \cdot |F_1| \cdot |F_2|$.

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Encode X_2 as a **stack** structure $\Rightarrow$

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Use classical reachability algorithm [SAS'03].